MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2017/2018

EME4076 – MECHANICAL VIBRATIONS
(ME)

7 MARCH 2018
09:00 a.m. – 11:00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This Question paper consists of 9 pages including cover page with 5 Questions and 3 appendices.

2. Attempt **FOUR** out of **FIVE** questions. All questions carry equal marks and the distribution of the marks for each question is given.

3. Please write all your answers in the Answer Booklet provided.

Question 1

Figure Q1 shows the essential elements of an instrument for recording vibrations, which consist of a frame and a light rigid rod that pins at O. The rod has a mass m attached to its free end and is held in a horizontal position by a spring of stiffness k, while its vibrations are damped by a viscous damper whose damping coefficient is c. The base of the instrument is fixed to a horizontal surface of an unbalanced motor which generates a displacement of $y_B = b \sin \omega t$. The vibration of the motor is recorded on the rotating drum of the instrument by a pen attached to the mass of the rod.

(a) Derive the equation of motion of the system.

[8 marks]

(b) Determine the maximum amplitude of the steady-state vertical displacement of the tip of the pen.

[9 marks]

(c) Determine the range of the spring constant k over which the magnitude of the recorded pen tip displacement is less than $1.5b$. It is knowing that the ratio ω/ω_n must be greater than unity.

[8 marks]

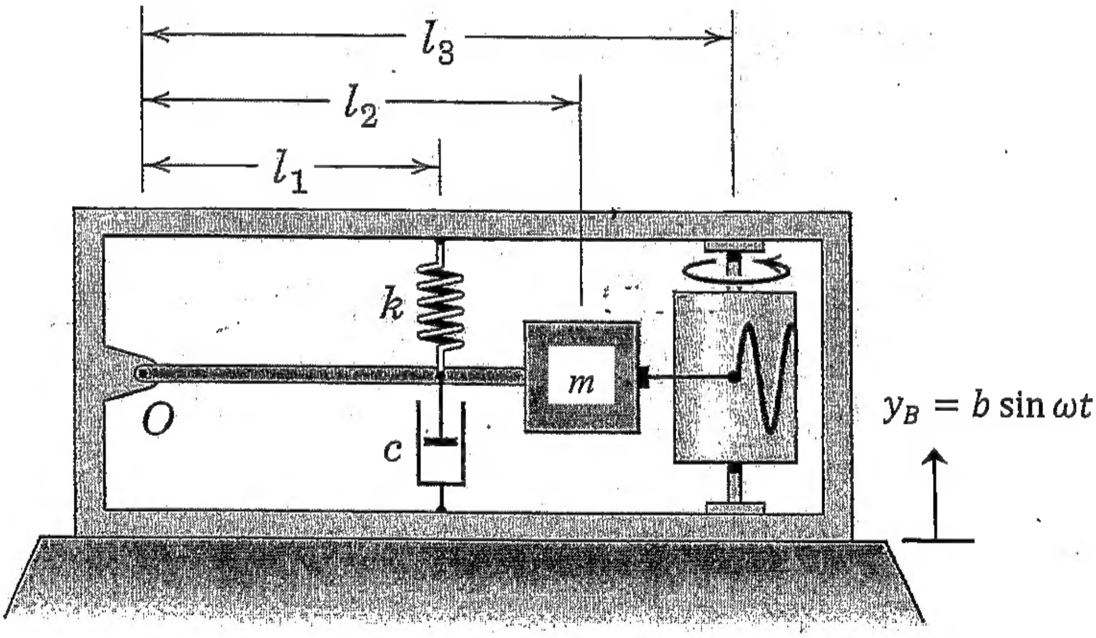

Figure Q1

Continued ....

Question 2

(a) A fragile instrument of mass $m = 100$ kg is used on a table that is vibrating because of its proximity to rotating machines that are running in the area. Since the vibration of the table affects the operation of the instrument, it is desired to use an accelerometer and spectrum analyzer to obtain the values of the frequency components of the acceleration (in g's) of the table. The acceleration of the table is shown in Figure Q2(a)(i). To reduce the acceleration of the instrument to an acceptable limit of $0.02g$'s. it is proposed to isolate the instrument from the table by means of several 25-mm-thickness rubber pads as shown in Figure Q2(a)(ii). Determine the maximum number of pads, which are arranged in parallel, required to reduce the acceleration of instrument to $0.02g$'s or less if the stiffness of each pad is 250 N/cm. Ignore the damping of each pad, since it is very small.

[13 marks]

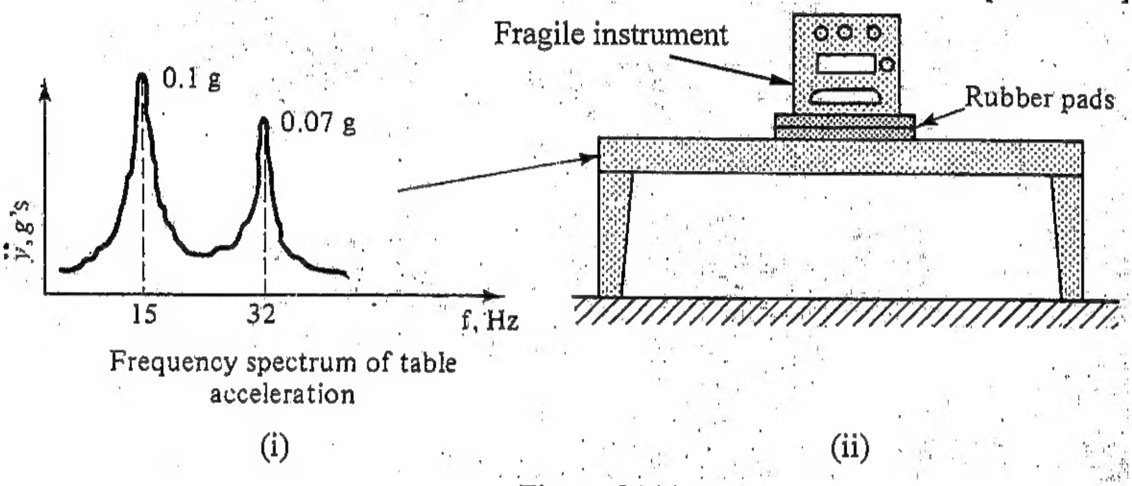

Figure Q2(a)

(b) A machine which has a total mass of 100 kg is supported on springs with total stiffness of 700 kN/m and a damper with damping ratio ς of 0.20. When the machine is operating at a speed of 3000 rpm, the unbalance force is found to be 350 N. Find the amplitude of vibration, the transmissibility and the force transmitted to the foundation of the system due to the unbalance at the operating speed.

[12 marks]

Continued ....

Question 3

Figure Q3 shows a machine with mass m_1 is bolted to the foundation through an undamped isolator with stiffness k_1. The machine is operated at a constant speed ω and is found to have an unbalanced mass m_0 located at a distance r from the center of rotation of the machine. To quell vibration due to the unbalanced mass, an absorber (spring-mass system) with stiffness k_2 and mass m_2 is attached to the machine.

(a) Derive the equations of motion of the system.

[10 marks]

(b) Determine the amplitudes of vibration for the machine m_1 and the absorber m_2.

[10 marks]

(c) Determine the required stiffness k_2 of the absorber such that the absorber is able to quell the amplitude of vibration of the machine m_1 to zero. What will be the amplitude of vibration for the absorber m_2 for this case?

[5 marks]

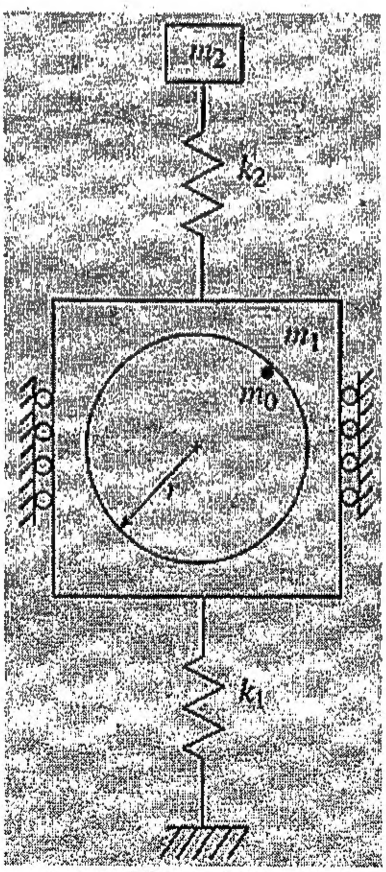

Figure Q3

Continued ....

Question 4

(a) As shown in Figure Q4(a), an elevator weighing $W = 35.6$ kN is attached to a steel cable that is wrapped around a drum rotating with a constant angular velocity of 3 rad/s. The radius of the drum is 0.3 m. The cable has a cross-sectional area of 6.45×10^{-4} m² and an effective modulus of elasticity of $E = 8.29 \times 10^{10}\ \mathrm{N}/m^2$. A malfunction in the motor drive system of the drum causes the drum to stop suddenly when the elevator is moving down and the length l of the cable is 15 m. Assuming the damping ratio of the cable is $\zeta = 0.05$. Determine the maximum stress σ_{max} in the cable.
(**Hint:** stress $\sigma = \text{tension in the cable/cross} - \text{sectional area of the cable}$)

[15 marks]

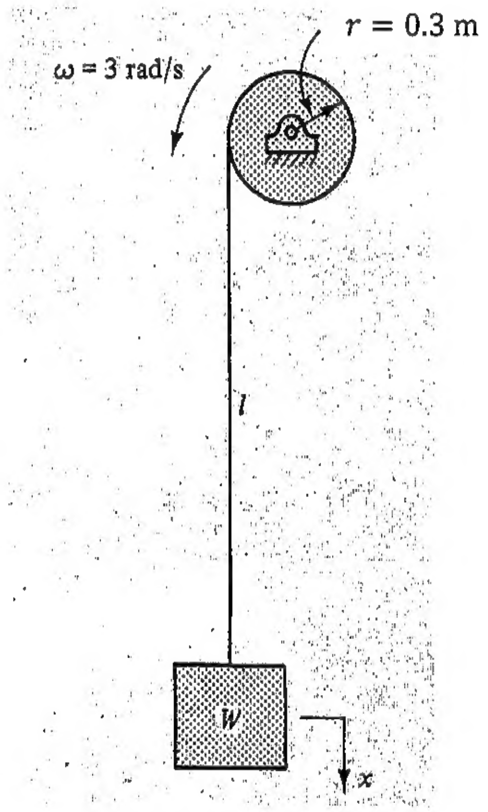

Figure Q4(a)

(b) If the elevator of part (a) is moving upward instead of downward as in part (a) when the drum suddenly stops at the instant $l = 15$ m, determine the time at which the stress σ in the cable is zero. Calculate the time to the accuracy of hundredth.
(**Hint:** the time is less than 0.025 s)

[10 marks]

Continued ....

Question 5

Figure Q5(i) shows a person carrying a precision instrument of mass m, rides in the elevator of a building in a standing position. The elevator, while moving with velocity v_0 at time $t = 0$, decelerates to zero velocity (stops) in time $t = t_1$ as shown in Figure Q5(ii). Assuming that the stiffness of the person in standing is k.

(a) Show that the equation of motion of the system is given as

$$m\ddot{z} + kz = -m\ddot{y}$$

where $z = x - y$, x and y are the displacements of the instrument and the elevator, respectively.

[5 marks]

(b) Determine the response of the system z for the time $t < t_1$.

[10 marks]

(c) Show that the maximum displacement z_{max} of the system for the time $t < t_1$ is

$$z_{max} = \frac{v_0}{\omega_n^2 t_1}\left[1 + \sqrt{1 + (\omega_n t_1)^2}\right]$$

[10 marks]

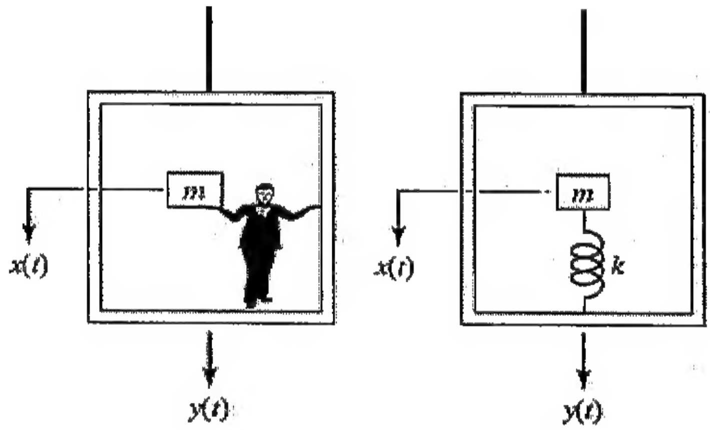

Figure Q5(i)

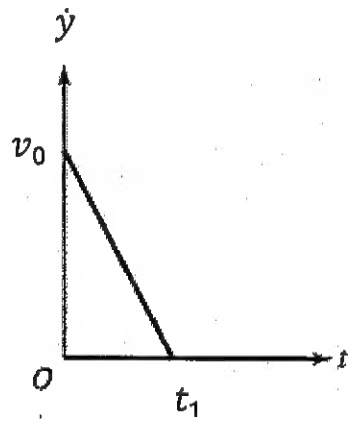

Figure Q5(ii)

Continued ....

Appendix 1

Vibrations

Equations of Motion

$$m\ddot{x} + c\dot{x} + kx = F_0 \sin\omega t$$

General Solution

$$x(t) = x_c(t) + x_p(t)$$

$$x_p(t) = D\sin(\omega t - \varphi)$$

$$D = \frac{F_0/k}{\sqrt{[1-r^2]^2 + (2\zeta r)^2}} \qquad where \quad r = \frac{\omega}{\omega_n}$$

$$\varphi = \tan^{-1}\frac{2\zeta r}{1-r^2}$$

$$x_c(t) = D_1 e^{\lambda_1 t} + D_2 e^{\lambda_2 t} \qquad \zeta > 1$$

$$x_c(t) = (B + Ct)e^{-\omega_n t} \qquad \zeta = 1$$

$$x_c(t) = e^{-\zeta\omega_n t}(B\cos\omega_d t + C\sin\omega_d t) \qquad \zeta < 1$$

$$\omega_n = \sqrt{k/m} = 2\pi f_n, \qquad f_n = \frac{1}{\tau_n} = \frac{\omega_n}{2\pi}$$

$$\zeta = \frac{c}{2m\omega_n}$$

$$\lambda_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1} \qquad \zeta \geq 1$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2} \qquad \zeta < 1$$

Force and Displacement Transmissibility

$$\frac{F_T}{F_0} = \frac{X}{Y} = \left[\frac{1 + (2\zeta r)^2}{(1-r^2)^2 + (2\zeta r)^2}\right]^{\frac{1}{2}}$$

Rotating Unbalance: Amplitude of Response

$$X = \frac{m_0 e}{m + m_0}\frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

Fourier's series

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}[a_n\cos n\omega t + b_n \sin n\omega t]$$

$$a_n = \frac{2}{\tau}\int_0^{\tau} x(t)\cos n\omega t\, dt \qquad b_n = \frac{2}{\tau}\int_0^{\tau} x(t)\sin n\omega t\, dt$$

Unit response function (underdamped)

$$h(t-\tau) = \frac{e^{-\xi\omega_n(t-\tau)}}{m\omega_d}\sin\omega_d(t-\tau)$$

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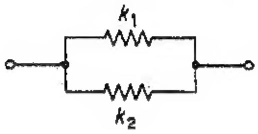
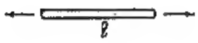
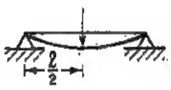
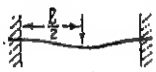
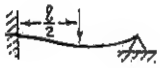
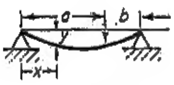
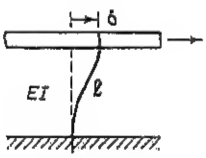
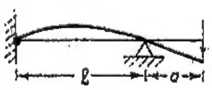
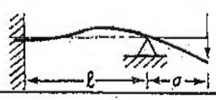
Appendix 2

Table of Spring Stiffness

	$k = \dfrac{1}{1/k_1 + 1/k_2}$	
	$k = k_1 + k_2$	
	$k = \dfrac{EI}{l}$	I = moment of inertia of cross-sectional area l = total length
	$k = \dfrac{EA}{l}$	A = cross-sectional area
	$k = \dfrac{GJ}{l}$	J = torsion constant of cross section
	$k = \dfrac{Gd^4}{64nR^3}$	n = number of turns
	$k = \dfrac{3EI}{l^3}$	k at position of load
	$k = \dfrac{48EI}{l^3}$	
	$k = \dfrac{192EI}{l^3}$	
	$k = \dfrac{768EI}{7l^3}$	
	$k = \dfrac{3EIl}{a^2b^2}$	$y_x = \dfrac{Pbx}{6EIl}(l^2 - x^2 - b^2)$
	$k = \dfrac{12EI}{l^3}$	
	$k = \dfrac{3EI}{(l+a)a^2}$	
	$k = \dfrac{24EI}{a^2(3l+8a)}$	

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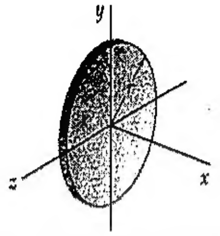
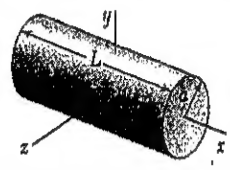
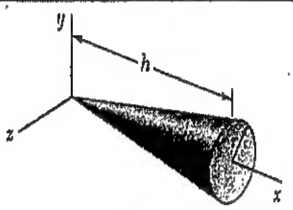
Appendix 3: Mass Moment of Inertia of Homogenous Solids



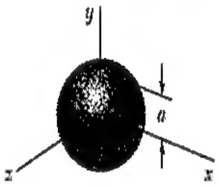
$I_x = \frac{1}{2}mr^2$
$I_y = I_z = \frac{1}{4}mr^2$



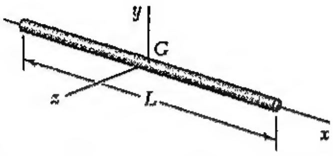
$I_x = \frac{1}{2}ma^2$
$I_y = I_z = \frac{1}{12}m(3a^2 + L^2)$



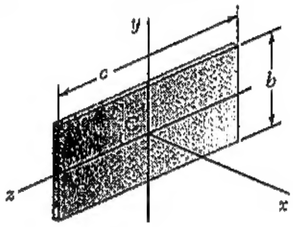
$I_x = \frac{3}{10}ma^2$
$I_y = I_z = \frac{3}{5}m(\frac{1}{4}a^2 + h^2)$



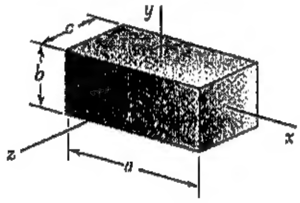
$I_x = I_y = I_z = \frac{2}{5}ma^2$



$I_y = I_z = \frac{1}{12}mL^2$



$I_x = \frac{1}{12}m(b^2 + c^2)$
$I_y = \frac{1}{12}mc^2$
$I_z = \frac{1}{12}mb^2$



$I_x = \frac{1}{12}m(b^2 + c^2)$
$I_y = \frac{1}{12}m(c^2 + a^2)$
$I_z = \frac{1}{12}m(a^2 + b^2)$

End of Paper